METHOD OF TEMPERATURE MEASUREMENT WITH THERMISTOR IN BRIDGE CIRCUIT WITH PULSE-VOLTAGE SUPPLY

S. B. Minkin

UDC 536.53

A method of temperature measurement with a semiconducting thermistor in the bridge circuit is considered which appreciably reduces the systematic error caused by heating of the probe by the test current.

Earlier studies of a thermistor energized from a pulse source have established the dependence of its heat load on the pulse duration and amplitude as well as on the duty cycle in the two cases of small current excursions [1] and large current excursions [2], having also provided a basis for the design of the measuring bridge so as to ensure a linear output signal without exceeding a prescribed maximum level of systematic measurement error [3]. Attempts to further reduce this error have resulted in a better method of measurement, called the "subtraction and extrapolation" method [4] and briefly described for application to an R_{Th}–R circuit. Its metrological advantages and simplicity of implementation, owing to widespread availability of microcomputer-based automatic data gathering and processing systems for experiments, make this method very effective where a thermistor successfully competes with other temperature transducers under certain operating conditions [5].

Here will be reported further studies of the subtraction-and-extrapolation method in application to a measuring bridge which contains a semiconducting thermistor as sensing element (Fig. 1).

The gist of this method is that two consecutive readings of the bridge output signal are taken separated by some time interval within the duration of one supply voltage pulse and the first reading is subtracted from the second one, whereupon the difference is subtracted from the first reading.

In order to explain the principle of this method, we will assume small excursions so that the linear model of a thermistor will be applicable. It is well known that, in the linear approximation, the bridge unbalance voltage depends on the excursion of the thermistor resistance from its value during balance according to the relation [6]

$$\Delta U_{\text{out}} = I_{\text{Th}0}\frac{R_2}{R_{\text{Th}0}+R_2}\Delta R_{\text{Th}} \tag{1}$$

The same condition of linearity yields

$$\Delta R_{\text{Th}} = -\beta_{\text{Th}0} R_{\text{Th}0}\Delta T, \tag{2}$$

so that

$$\Delta U_{\text{out}} = -I_{\text{Th}0}\frac{R_2}{R_{\text{Th}0}+R_2}\beta_{\text{Th}0} R_{\text{Th}0}\Delta T. \tag{3}$$

According to relation (3), an analysis of the synthematic error on the basis of the bridge unbalance output signal requires determination of the rise of the thermistor temperature ΔT produced by the sequence of supply voltage pulses. For this determination we will use the structural diagram of a bridge with thermistor and independent voltage supply (no feedback) [6] (Fig. 2), according to which the bridge has a transfer function with respect to temperature

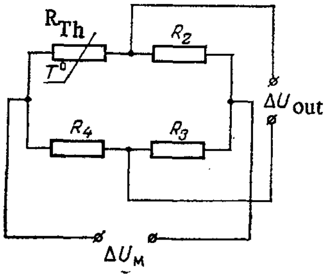
$$\frac{\Delta T(p)}{\Delta U_{\text{M}}(p)} = \frac{2U_{\text{Th}0}}{k(1-D_0\delta)(R_{\text{Th}0}+R_2)(\tau_0 p+1)}. \tag{4}$$

A comparison of transfer function (4) with the analogous transfer function of the R_{Th}–R circuit [1] reveals that they are identical. This suggests that the temperature rise of the

---

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 47, No. 6, pp. 959-964, December, 1984. Original article submitted December 26, 1983.

0022-0841/84/4706-1439$08.50



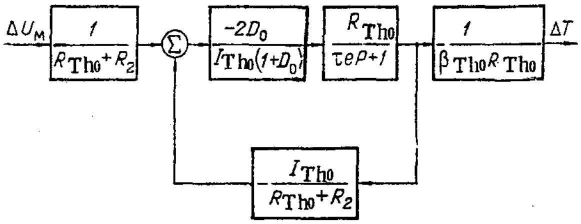


Fig. 2

Fig. 1. Schematic diagram of measuring bridge.

Fig. 2. Structural diagram of measuring bridge with semiconducting thermistor and without feedback [6].

thermistor in the bridge circuit can be determined with the aid of solutions [1] for the R_{Th}–R circuit.

During a pulse $(0 \leqslant \varepsilon \leqslant \gamma)$ we have

$$\Delta T[n,\ \varepsilon] = \frac{2\Delta U_{M} U_{Th0}}{k(1-D_0\delta)(R_{Th0}+R_2)}\left\{1-\frac{e^{-\alpha\varepsilon}}{1-e^{-\alpha}}\left[(1-e^{-\alpha(1-\gamma)})-(1-e^{\alpha\gamma})e^{-\alpha(n+1)}\right]\right\}. \tag{5}$$

During a pause $(\gamma \leqslant \varepsilon \quad 1)$ we have

$$\Delta T[n,\ \varepsilon] = \frac{2\Delta U_{M} U_{Th0}}{k(1-D_0\delta)(R_{Th0}+R_2)}\,\frac{(1-e^{\alpha\gamma})}{1-e^{-\alpha}}\left[e^{-\alpha(n-1)}-1\right]e^{-\alpha\varepsilon}. \tag{6}$$

The equations of the envelopes are respectively

$$\Delta T_{max} = \Delta T[n,\ \gamma] = \frac{2\Delta U_{M} U_{Th0}}{k(1-D_0\delta)(R_{Th0}+R_2)}\left\{\frac{(1-e^{-\alpha\gamma})\left[1-e^{-\alpha(n+1)}\right]}{1-e^{-\alpha}}\right\}, \tag{7}$$

$$\Delta T_{min} = \Delta T[n,\ 0] = \frac{2\Delta U_{M} U_{Th0}}{k(1-D_0\delta)(R_{Th0}+R_2)}\left\{\frac{(e^{\alpha\gamma}-1)(1-e^{-\alpha n})}{1-e^{-\alpha}}\right\}e^{-\alpha}. \tag{8}$$

The maximum and minimum temperature rises of the thermistor in the state of dynamic equilibrium are

$$\Delta T_{max.s.s} = \Delta T[\infty,\ \gamma] = \frac{2\Delta U_{M} U_{Th0}}{k(1-D_0\delta)(R_{Th0}+R_2)}\,\frac{(1-e^{-\alpha\gamma})}{1-e^{-\alpha}}, \tag{9}$$

$$\Delta T_{min.s.s} = \Delta T[\infty,\ 0] = \frac{2\Delta U_{M} U_{Th0}}{k(1-D_0\delta)(R_{Th0}+R_2)}\,\frac{(e^{\alpha\gamma}-1)}{1-e^{-\alpha}}. \tag{10}$$

Inserting the expressions for ΔT from (5)-(10) into relation (3), taking into account that $U_{Th_0} I_{TH_0} = P_{Th_0}$ and $D_0 = -P_{Th_0}\beta_{Th_0}/k$ [6], we obtain expressions for the bridge unbalance signal during a pulse and during a pause, respectively, as well as equations of the envelopes and the maximum unbalance in the quasisteady state

$$(0 \leqslant \varepsilon \leqslant \gamma):$$

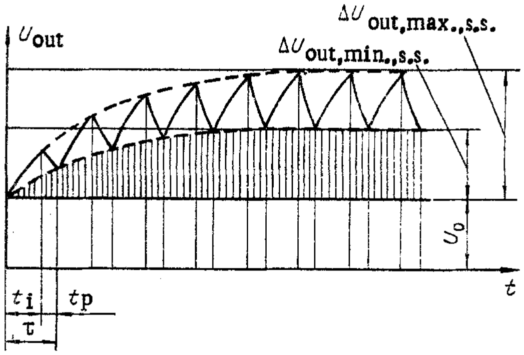
$$\Delta U_{out}[n,\ \varepsilon] = A\left\{1-\frac{e^{-\alpha\varepsilon}}{1-e^{-\alpha}}\left[1-e^{-\alpha(1-\gamma)}-(1-e^{\alpha\gamma})e^{-\alpha(n+1)}\right]\right\}, \tag{11}$$

$$(\gamma \leqslant \varepsilon \leqslant 1):$$

$$\Delta U_{out}[n,\ \varepsilon] = A\,\frac{(1-e^{\alpha\gamma})}{1-e^{-\alpha}}\left[e^{-\alpha(n-1)}-1\right]e^{-\alpha\varepsilon}, \tag{12}$$

$$\Delta U_{out.max} = \Delta U_{out}[n,\ \gamma] = A\,\frac{(1-e^{-\alpha\gamma})}{1-e^{-\alpha}}\left[1-e^{-\alpha(n+1)}\right], \tag{13}$$

$$\Delta U_{out.min} = \Delta U_{out}[n,\ 0] = A\,\frac{(e^{\alpha\gamma}-1)}{1-e^{-\alpha}}(1-e^{-\alpha n})e^{-\alpha}, \tag{14}$$



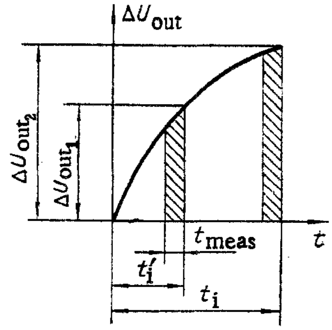


Fig. 3 Fig. 4

Fig. 3. Transient process in circuit of measuring bridge with pulse-voltage supply.

Fig. 4. Timing diagram of output signal measurement.

$$\Delta U_{out.\,max.\,s.s} = \Delta U_{out}\,[\infty,\ \gamma] = A\,\frac{(1-e^{-\alpha\gamma})}{1-e^{-\alpha}}, \tag{15}$$

$$\Delta U_{out.\,min.\,s.s} = \Delta U_{out}\,[\infty,\ 0] = A\,\frac{(e^{\alpha\gamma}-1)}{1-e^{-\alpha}}\,e^{-\alpha}. \tag{16}$$

In expressions (11)-(16)

$$A = \frac{2\Delta U_M R_2 R_{Th0} D_0}{(1-D_0\delta)(R_{Th0}+R_2)^2}.$$

In the conventional method of measurement, with the output signal read only once within the duration of a supply voltage pulse, Eq. (13) of the envelope characterizes the upper values of the systematic error during the transient period and, with the measured temperature remaining constant, this error reaches its maximum when the measuring circuit is in the dynamic equilibrium state (Eq. (15), Fig. 3).

According to the proposed method, we perform the aforementioned operations of subtraction of two output signals read during one pulse of the supply voltage. Let the first reading be taken at the end of interval t_i', the latter determined by the relative time $\varepsilon < \gamma$, and the second reading be taken at the end of the given supply voltage pulse t_i so that $\varepsilon = \gamma$. It will be further assumed that the reading time is much shorter than the two intervals t_i' and $(t_i - t_i')$ (Fig. 4). Then $\Delta U_{out,1} = \Delta U_{out}[n,\varepsilon]$ and $\Delta U_{out,2} = \Delta U_{out}[n,\gamma]$. The difference between second reading and first reading is

$$\Delta U_{out\,2} - \Delta U_{out\,1} = \Delta U_{out}\,[n,\ \gamma] - \Delta U_{out}\,[n,\ \varepsilon] = \Delta.$$

The magnitude of the resultant signal is

$$\Delta U_{out_1} - \Delta = 2\Delta U_{out}\,[n,\ \varepsilon] - \Delta U_{out}\,[n,\ \gamma]. \tag{17}$$

We first consider the general case where $0 < n < \infty$. This case corresponds to Eqs. (11) and (13), insertion of which into relation (17) yields, after some transformation,

$$2\Delta U_{out}\,[n,\ \varepsilon] - \Delta U_{out}\,[n,\ \gamma] = A\left\{\frac{(e^{\alpha\gamma}-1)(1-e^{-\alpha n})}{1-e^{-\alpha}}\,e^{-\alpha}(2e^{-\alpha\varepsilon}-e^{-\alpha\gamma}) + (1-2e^{-\alpha\varepsilon}+e^{-\alpha\gamma})\right\}. \tag{18}$$

Analogously, for the dynamic equilibrium state ($n \to \infty$) insertion of expressions (11) and (15) into relation (17) yields in the final form

$$2\Delta U_{out}\,[\infty,\ \varepsilon] - \Delta U_{out}\,[\infty,\ \gamma] = A\left[\frac{(e^{\alpha\gamma}-1)\,e^{-\alpha}}{1-e^{-\alpha}}(2e^{-\alpha\varepsilon}-e^{-\alpha\gamma}) + (1-2e^{-\alpha\varepsilon}+e^{-\alpha\gamma})\right]. \tag{19}$$

Upon an examination of relations (18) and (19), it becomes evident that with

$$2e^{-\alpha\varepsilon} - e^{-\alpha\gamma} = 1 \tag{20}$$

they become respectively Eq. (14) of the lower envelope and expression (16) for the minimum steady-state value of the output signal in the dynamic equilibrium state.

With the aid of equality (20) one can determine the time shift between the beginnings of two measurements which will yield this result. Replacing here relative time with absolute time, we obtain, after some transformation,

$$\Delta t = t_i' = -\tau_\theta \ln \frac{1 + e^{-t_i/\tau_\theta}}{2}. \tag{21}$$

It thus follows that application of the described method of temperature measurement so as to satisfy the time relations (21) reduces the systematic error, which is here determined by the lower envelope of the transient and not conventionally by its upper envelope.

NOTATION

ΔU_{out}, unbalance voltage of the measuring bridge circuit; ΔU_M, amplitude of a supply voltage pulse; R_2, R_3, R_4, resistances in three arms of the bridge; R_{Th}, thermistor resistance; ΔR_{Th}, excursion of the thermistor resistance from its value during balance; R_{Th0}, I_{Th0}, β_{Th0}, thermistor resistance, current, and temperature coefficient of resistance during balance of the measuring circuit; ΔT, temperature rise of the thermistor as a result of heating by the test current; k, statistical dispersion of the thermistor; $\delta = (R_{Th0} - R_2)/(R_{Th0} + R_2)$, dimensionless resistance parameter; D_0, dynamic coefficient [6]; $\gamma = t_i/\tau$, duty cycle of supply voltage pulses; t_i, pulse duration, t_p, pause duration; $\tau = t_i + t_p$, pulse repetition period; n, arbitrary pulse repetition period; $\varepsilon = \Delta t/\tau$, relative time varying from 0 to 1; Δt, time elapsed since the beginning of each pulse; $\tau_\theta = \tau_0/(1 - D\delta)$, time constant of the measuring circuit; τ_0, time constant of the thermistor; $\alpha = \tau/\tau_\theta$; U_0, bridge unbalance voltage dependent on the measured temperature; P_{Th0}, power dissipated by the thermistor during balance of the measuring circuit; t_i', time interval at the end of which the first reading of the output signal is taken; t_{meas}, time of an output signal measurement; p, Laplace variable.

LITERATURE CITED

1. S. B. Minkin, A. G. Shashkov, and V. E. Ulashchik, "Thermal state of thermistor in R_{Th}–R circuit energized by voltage pulses," Inzh.-Fiz. Zh., 30, No. 6, 1081-1088 (1976).
2. S. B. Minkin, A. G. Shashkov, et al., "Pulse mode of R_{Th}–R circuit operation with large current excursions," Izv. Akad. Nauk BSSR, Ser. Fiz.-Energ. Nauk, No. 3, 128-134 (1976).
3. S. B. Minkin et al., "Design method for measuring bridge circuit with semiconducting thermistor and pulse-voltage supply," Izmer. Tekh., No. 1, 69-70 (1978).
4. S. B. Minkin et al., "Subtraction-and-extrapolation method of temperature measurement," Proc. All-Union Scientific Conf. on Methods and Means of Computer-Aided Diagnosis of Gas-Turbine Engines and Their Components [in Russian], Kharkov Institute of Aviation (1977), pp. 293-295.
5. S. B. Minkin and A. I. Brazgovka, "Device for measuring nonelectrical quantities," Inventor's Certificate No. 731317, Byull. Izobret., No. 16 (1980).
6. A. G. Shashkov, Thermistors and Their Applications [in Russian], Énergiya, Moscow (1967).